‘Unless’ and Precedence of Connectives

1. ‘Unless’

The translation of ‘unless’ is not straightforward. Here are two examples.

(A) ‘*Unless* Sue speaks to me and confirms the date, I cannot arrange to see Jack and this is not good’ (see Reader p. 13).
(B) ‘Plato sings *unless* we pay him.’

There are three possible interpretations, each of which yields a different translation into PL. You decide what looks most natural to you.

				①	②	③
Line	*P*	*Q*	*P unless Q*	$P \vee Q$	$P >\!\!-\!\!< Q$	$\sim Q \supset P$
1	T	T	?	T	F	T
2	T	F	T	T	T	T
3	F	T	T	T	T	T
4	F	F	F	F	F	F

① *Disjunction*. Proposition B expresses the idea that either Plato sings or we pay him (not to sing). So, *if* we do *not* pay him, he sings. (The ‘if’ and the ‘not’ are hints for ③.) So, in the truth table, ‘*P* unless *Q*’ should be true just in case ‘Plato sings’ is true and ‘We pay him’ is false (Line 2). Similarly, if Plato does not sing, then we probably pay him: ‘*P* unless *Q*’ is true when *P* is false and *Q* is true (Line 3). What about the sitation where Plato does *not* sing and we do *not* pay him (Line 4)? This is where the *conditional* connotations of ‘unless’ comes into play: if he does not sing, and we do not pay him, then our paying or not paying is irrelevant to his singing. So, ‘*P* unless *Q*’ is false when *P* and *Q* are both false.

The crucial situation is where Plato sings *and* we pay him (Line 1). We could accept that Plato sings *regardless* of our paying. Then we think that ‘*P* unless *Q*’ is true when *P* and *Q* are true. Therefore, our interpretation of ‘unless’ is that of an inclusive disjunction.

② *Contravalence*. The fact that he sings *and* we pay him (Line 1) means that our proposition ‘Plato sings *unless* we pay him’ has no force, or does not apply, just as in Line 4: either Plato sings or we pay him *and* it is *not* the case that he sings while (and) we pay him. So, we think that ‘*P* unless *Q*’ is false when both *P* and *Q* are true. And thus our interpretation of ‘unless’ is that of an exclusive disjunction (Reader p. 16): $P \vee Q \mathbin{\&} \sim(P \mathbin{\&} Q)$, which is equivalent to $P >\!\!-\!\!< Q$.

③ *Conditional*. Given the hint above, we translate ‘unless ϕ’ as ‘if not ϕ’: if we do *not* pay, Plato sings. So, *P unless Q* becomes $\sim Q \supset P$. (Note that the consequent goes first.) But the truth table above proves that this interpretation is *equivalent* to the disjunction $P \vee Q$. So we are back to reading ‘unless’ as ‘or’.

2. The Precedence and Brackets

Opinions divide over the precedence of the connectives. Some think & and ∨ trump ⊃ and ≡. But this may still not yield enough clarity: we need brackets anyway. Consider these cases:

	No Brackets	*With Brackets*	
(a)	*P* ∨ *Q* & *R*	(*P* ∨ *Q*) & *R*	*P* ∨ (*Q* & *R*)
		'Plato sings or the Queen smiles **and** the robot talks.'	'Plato sings **or** the Queen smiles and the robot talks.'

- Brackets bring out the main connective more clearly. Use them.
- Brackets change the meaning of the complex wff.
- Brackets change the truth value of the complex formula: the propositions are not equivalent, as we can prove with the following truth table.

P	*Q*	*R*	(*P* ∨ *Q*)	& *R*	*P* ∨	(*Q* & *R*)	
T	T	T	T	**T**	T	**T**	
T	T	F	T	**F**	T	**F**	
T	F	T	T	**T**	T	**F**	←
T	F	F	T	**F**	T	**F**	
F	T	T	T	**T**	T	**T**	
F	T	F	T	**F**	F	**F**	
F	F	T	F	**F**	F	**F**	
F	F	F	F	**F**	F	**F**	

(b)	*P* ⊃ *Q* & *Q* ⊃ *P*	*P* ⊃ (*Q* & *Q*) ⊃ *P*	(*P* ⊃ *Q*) & (*Q* ⊃ *P*)
		'If the Pope sings then if the Queens smiles and smiles then the Pope sings.'	'If the Pope sings then the Queen smiles, and if the Queen smiles then the Pope sings.'

- The first brackets merely emphasise the alleged precedence—but the proposition makes no sense.
- The second makes sense: it expresses equivalence (≡), *viz*. *P* if and only if *Q*, or *P* iff *Q*.