solutions

Grammar and Translations

1. Wffs (Well Formed Formulae)

Are these formulae well formed, i.e. grammatical in PL? If not, suggest corrections. (When finished, feel free to turn left and/or right to compare your results.)

(a) $P \,\&\, Q$ ✓

(b) $\& P\, Q$ ○

(c) $P \sim Q$ ○

(d) $P \vee Q \vee R \vee S \vee T$ ✓

(e) $P \supset Q \,\&\, R$ ✓ brackets?

(f) $Q \,\&\, P$ ✓

(g) $\sim P \vee \sim Q\, (\& R)$ ○

(h) $\sim\sim\sim\sim P$ ✓

(i) $P \supset Q \,\&\, \supset R \vee (S \vee T)$. ○

(j) $(P \supset Q) \,\&\, \sim P \vee S \vee \sim S \vee \sim Q$ ✓

(k) $P \,\&\, \sim P \sim Q \supset R$ ○

(l) $P \,\&\, \sim P \,\&\, \sim\sim P \,\&\, \sim\sim\sim P$ ✓

Think about. Are the ungrammatical formulae easy to identify? Why are the ungrammatical ones are ill formed? Are there any problematic cases that we need to discuss? Do all wffs actually mean anything?

2. Translating into PL

Identify the *basic* propositions, for instance by encircling or underlining them. Give them a PL name (i.e. an interpretation), such as P and Q. You may be inventive, but be as clear as possible. Then identify the logical connectives. Finally, write the complex propositions in logical notation.

(a) Jack swims and Sue wins the lottery.

$P \,\&\, Q$

(b) Ella is not at home and Dave is not reading a book.

$\sim P \,\&\, \sim Q$

(c) Jerry likes reading Hume or Jane likes reading Plato.

$P \vee Q$

(d) If there is smoke, (then) there is a fire.

$P \supset Q$

(e) If we look closely, (then) we see an elephant and we see a lion or (we see) a giraffe.

$P \supset Q \& R \vee S$ $\quad P \supset (Q \& R \vee S)$ or better: $(P \supset (Q \& R)) \vee S$ or $(P \supset Q) \& (R \vee S)$

(f) Fred plays the trumpet and the recorder. He does not play the guitar. But Freya or Bella do.

Fred – trumpet : P
Fred – recorder : Q
Fred – guitar : R
Freya – guitar : S
Bella – guitar : T

$(P \& Q \& \sim R) \& S \vee T$

(g) Maggie likes red or blue and Keith likes black, grey, and white.

Maggie – red : P
Maggie – blue : Q
Keith – black : R
Keith – grey : S
Keith – white : T

$(P \vee Q) \& (R \& S \& T)$

(h) It is not the case that dogs bite and cats do not.

$\sim (P \& \sim Q)$

(i) If John is angry, (his cheeks are red and his breath is shallow.)

$P \supset (Q \& R)$

(j) If Clare is not at home, (then) her cat is not fed and the plants are not watered. But the cat is fed and the plants are watered. So, Clare is at home.

$(\sim P \supset (\sim Q \& \sim R)) \& (Q \& R) \& P$ or

$\sim P \supset \sim (Q \& R)$
$Q \& R$
P

(k) If the pizza is delicious or stunning, and Rob made it, then he ought to feel proud. If the pizza is burnt and bland, he ought not to feel proud. modality

Pizza – delicious : P
Pizza – stunning : Q
Rob – Pizza : R
Rob – proud : S
Pizza – burnt : T
Pizza – bland : U

$((P \vee Q \& R) \supset S) \& (T \& U) \supset \sim S$

Think about. How much must you interpret the statements? Are some of them ambiguous? Do brackets help in these cases? There is actually one argument: can you identify it, and is it valid?