Propositional Logic III

1. A detective establishes these true facts:
   The window is open.
   The door is locked and the window is closed if and only if Sue is out.
   The key is under the mat if and only if Sue is out.
   Sue is at home if and only if the door is locked and the window is open.
   Where is Sue? Where is the key? And is the door locked? Devise your own translations.[1]

2. Formalise these statements. Key: S = Spinoza is a philosopher; H = Hume is a philosopher; L = Leibniz is a philosopher, B = Berkeley is a philosopher.[2]
   (a) If either Hume or Leibniz is a philosopher, so is Spinoza.
   (b) Hume is a philosopher if and only if Berkeley is not.
   (c) If Spinoza is a philosopher if and only if Hume is, then Berkeley is one too.
   (d) If Hume is no philosopher, then neither are Leibniz and Spinoza.
   (e) Leibniz and Berkeley are both philosophers only if Hume and Spinoza are.

3. Which of these PL sentences are contradictions, which are tautologies, and which are neither?[3]
   (a) $P \mathbin{\&} ((\sim P \vee Q) \mathbin{\&} \sim Q)$
   (b) $(Q \mathbin{\&} (\sim\sim P \vee R)) \vee \sim(P \mathbin{\&} Q)$
   (c) $(P \vee (Q \mathbin{\&} \sim R)) \vee \sim((\sim P \vee R) \vee Q)$

4. Show with truth tables whether these arguments are valid.[4]
   (a) $P,\ P \supset Q,\ Q \supset R \therefore R$
   (b) $\sim R,\ P \supset R,\ \sim P \supset Q \therefore Q$
   (c) $\sim(P \equiv (Q \mathbin{\&} R)),\ S \vee \sim Q,\ \therefore \sim(S \supset P)$

5. Are the following sequents valid? Show by natural deduction.
   (a) '$\sim P \supset Q$' $\vdash$ '$P \vee Q$'
   (b) '$(P \mathbin{\&} \sim Q) \supset R$', '$\sim R$', '$P$' $\vdash$ 'Q'
   (c) '$(Q \supset R) \mathbin{\&} (S \supset T)$', '$(U \supset V) \mathbin{\&} (W \supset X)$', '$Q \vee U$' $\vdash$ $R \vee V$

1 Suggested interpretation: P = The key is under the mat; Q = Sue is out; R = The door is locked; S = The window is closed. Remember: all propositions are true together.
2 After Smith, P. (2003). *Formal Logic*. Cambridge: Cambridge University Press (ch. 14).
3 Smith, *op. cit.*, p. 106.
4 The '∴' indicates the inference in PL.