

## Propositional Logic III

5. Are the following sequents valid? Show by natural deduction.

(a)  $\sim P \supset Q \vdash P \vee Q$

1	(1)	$\sim P \supset Q$	Premise
2	(2)	$\sim(P \vee Q)$	Assumption
3	(3)	$P$	Assumption
3	(4)	$P \vee Q$	3 $\vee$ I
2, 3, 4	(5)	$(P \vee Q) \& \sim(P \vee Q)$	2, 4 $\&$ I
1	(6)	$\sim P$	3, 5 $\sim$ I
1	(7)	$Q$	1, 6 $\supset$ E
1	(8)	$P \vee Q$	7 $\vee$ I
1, 2	(9)	$(P \vee Q) \& \sim(P \vee Q)$	2, 8 $\&$ I
1, 2	(10)	$\sim Q$	8, 9 $\sim$ I
1, 2	(11)	$Q \& \sim Q$	7, 10 $\&$ I
1	(12)	$\sim\sim(P \vee Q)$	2, 11 $\sim$ I
1	(13)	$P \vee Q$	12 $\sim$ E

(b)  $(P \& \sim Q) \supset R, \sim R, P \vdash Q$

1	(1)	$(P \& \sim Q) \supset R$	Premise
2	(2)	$\sim R$	Premise
3	(3)	$P$	Premise
4	(4)	$\sim Q$	Assumption
3, 4	(5)	$P \& \sim Q$	3, 4 $\&$ I
1, 3, 4	(6)	$R$	1, 5 $\supset$ E
1, 2, 3, 4	(7)	$R \& \sim R$	2, 6 $\&$ I
1, 2, 3, 4	(8)	$\sim(P \& \sim Q)$	5, 7 $\sim$ I
1, 2, 3, 4	(9)	$\sim(P \& \sim Q) \& (P \& \sim Q)$	5, 8 $\&$ I
1, 2, 3	(10)	$\sim\sim Q$	8, 9 $\sim$ I
1, 2, 3	(11)	$Q$	10 $\sim$ E

(c) PTO

(c)  $(Q \supset R) \& (S \supset T)$ ,  $(U \supset V) \& (W \supset X)$ ,  $Q \vee U \vdash R \vee V$

1	(1)	$(Q \supset R) \& (S \supset T)$	Premise
2	(2)	$(U \supset V) \& (W \supset X)$	Premise
3	(3)	$Q \vee U$	Premise
1	(4)	$Q \supset R$	1 &E
5	(5)	$Q$	Assumption
1, 5	(6)	$R$	4, 5 $\supset$ E
1, 5, 6	(7)	$R \vee V$	6, $\vee$ I
2	(8)	$U \supset V$	2 &E
9	(9)	$U$	Assumption
2, 9	(10)	$V$	8, 9 $\supset$ E
2, 9, 10	(11)	$R \vee V$	10 $\vee$ I
1, 2, 3	(12)	$R \vee V$	3, 5, 6, 7, 9, 10, 11 $\vee$ E

