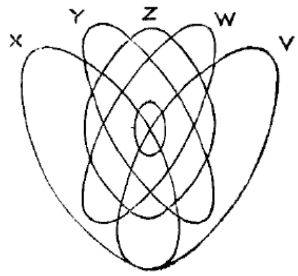
Predicate Logic I

‘I think, therefore I am’: a Syllogism? Descartes: “When someone says ‘I am thinking, therefore I am, or I exist’, he does not deduce existence from thought by means of a syllogism, but recognises it as something self-evident by a simple intuition of the mind. This is clear from the fact that if he were deducing it by means of a syllogism, he would have to have previous knowledge of the major premise ‘Everything which thinks is, or exists’; yet in fact he learns it from experiencing in his own case that it is impossible that he should think without existing.” (*Second Replies* AT VII 140–1).

The Limits of Syllogistics.



(i) More than three terms lead to unwieldy diagrams.[1] (ii) Categorial statements are too limited to accommodate propositions with a complex *interal structure* or *mixed quantifiers*, e.g., ‘If all dualists who went to the conference accept utilitarianism, then some dualists do not accept utilitarianism’.

“Logic is an old subject, and since 1879 is has been a great one.”[2] That year, Gottlob Frege (1848–1925) published the *Begriffsschrift*, which was a paradigm shift in formal logic. In syllogistics ‘All humans are mortal’ is taken in the form of *subject* and predicate. Frege’s insight: ‘all humans’ is a peculiar ‘subject’, which has a different semantic role than a name, such as ‘Socrates’. The proposition is more like a complex of *two* related *functions*, *viz*. ‘(…) is human’ and ‘(…) is mortal’, such that whatever satisfies (or is true of) the former satisfies the latter. Predicate logic allows for substructures, as in ‘Hume rejects everything that some philosophers say’.

Approaching QL

QL is the language for quantified propositions, or the language of predicate logic. In a rough and ready slogan: PL + syllogistic quantification (‘all’, ‘some’) = QL.

Syntax of QL

(1) *Predicates*: F, G, H, etc. stand for properties and relations. Monadic Predicates take one argument, e.g., (…) *is smart*. Polyadic Predicates take two or more arguments, e.g., (…) *loves* (…), or (…) *stands between* (…) *and* (…).
(2) *Individual Constants*: *a*, *b*, *c*, etc. are like names that pick out specific individuals.
(3) *Sentences*: F*a*, *Ga*, *Rab*, *Rabc*, etc. For example, the atomic sentence *Fa* means ‘Juliet is beautiful’, and *Rba* means ‘Romeo loves Juliet’. Sentences with individual constants are *closed*.
(4) *Connectives*, as in PL: ~, &, ∨, ⊃, ≡, e.g., (*Fa* ⊃ *Rba*), (*Rba* & ~*Rab*).

1 Fig. from Venn, J. (1880). On the Diagrammatic and Mechanical Representation of Propositions and Reasonings. *Philosophical Magazine*, *Series 5*, *Vol. 9*, p. 7.
2 Quine, W. V. O. (1962). *Methods of Logic*. London: Routledge (p. vii).

(5) *Individual Variables*: x, y, z, etc., also called ‘arguments’ or ‘designators’. For example, Fx says no specific individual is beautiful; x holds the place for one that could be (like pronoun). Hence Fx (or ϕv) is a propositional function. Formulae with individual variables are *open*.

(6) *Quantifiers*: $\forall x$, $\exists x$, or with optional brackets $(\forall x)$, $(\exists x)$.

- Existential quantifier: $\exists x$ means there is at least one x, or some x (cf. SiP). So, $\exists xFx$ says ‘there is at least one individual x which is/has F’.
- Universal quantifier: $\forall x$ means for all x, or every x (cf. SaP). So, $\forall xFx$ says ‘every individual x has/is F’.
- Interrelations: $\exists v\phi v \equiv {\sim}\forall v{\sim}\phi v$ (where v and ϕ are used as metavariables).
- No need for ‘none’: $\forall v{\sim}\phi v$, or ${\sim}\exists v\phi v$.[3]
- Quantifiers refer to a *universe of discourse*, the background set of things we are talking about or ‘quantifying over’. The notion of a set is important for the semantics of QL.
- The order of quantifiers matters: given that F means ‘is happy’, x is a person and y is a time, (i) $\forall x\exists yFxy$ says that everybody is happy at some time, but (ii) $\exists y\forall xFxy$ says that at some time everybody is happy (i.e. fallacy of illicit quantifier shift).[4] Compare also: (iii) $\forall x\exists yLxy$ (everybody loves somebody) and (iv) $\exists y\forall xLxy$ (someone is loved by everyone)(see scope below).

(7) *‘Quantifying in’*: systematically replace constants with variables and prefix the formula with the quantifier of the same variable: (i) Rab, (ii) $\exists yRay$, (iii) $\forall x\exists yRxy$, which says, e.g., that everybody loves somebody. Example: ‘René is French and loves cheese’ can be formalised like this: Fa & Lac. To make this true about *every* French individual, we quantify in: $\forall x(Fx \,\&\, Lxc)$.

(8) *Scope*: what falls under the quantifier. In $\forall x(\exists yFy \supset \exists z(Fz \vee {\sim}Rxyz)$, the scope of $\forall x$ is the entire wff, the scope of $\exists y$ is just Fy, and the scope of $\exists z$ is $(Fz \vee {\sim}Rxyz)$. Example (6)(iii): $\forall x$ has $\exists yLxy$ in its scope, which is thus wider than $\exists y$’s scope. A variable is *bound* if inside the scope of the quantifier of the same variable, and otherwise *free*. For instance, in the sentence $\forall x(Lxy \,\&\, Fa)$, only x is bound (but y is inside the quantifier’s scope). Only the first occurrence of x is free in $Rxy \supset \exists x({\sim}Rxy \vee Lyx)$. Brackets are (again) crucial: (i) Both occurrences of x are bound in $\exists x(Fx \supset Gx)$; but (ii) $\exists xFx \supset Gx$ binds only the first occurrence of x. One central aim of QL is to avoid scope ambiguities.

(9) *Translations* from syllogistics: (i) SaP ≈ $\forall x(Sx \supset Px)$; (ii) SiP ≈ $\exists x(Sx \,\&\, Px)$; (iii) SeP ≈ $\forall x(Sx \supset {\sim}Px)$; (iv) SoP ≈ $\exists x(Sx \,\&\, {\sim}Px)$.[5]

“For a ***complete*** logical argument,” Arthur began with admirable solemnity, “we need two prim Misses——”

“Of course!” she interrupted. “I remember that word now. And they produce—?”

“A Delusion,” said Arthur.

“Ye—es?” she said dubiously. “I don’t seem to remember that so well. But what is the ***whole*** argument called?”

“A Sillygism.”

3 Two further equivalences: (i) ${\sim}\forall v\phi v \equiv \exists v{\sim}\phi v$; (ii) ${\sim}\exists v{\sim}\phi v \equiv \forall v\phi v$.

4 But note that (iii) $\forall y\exists xFyx$ would mean the same as (i).

5 Snippet from Lewis Carroll, *Sophie and Bruno*, ch.18.