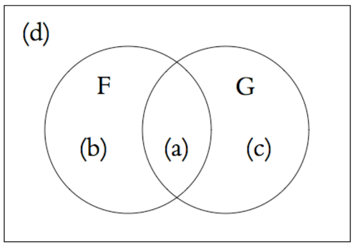
# Predicate Logic II

*Revisiting the connection to syllogistics.*
In syllogistics, SaP $\vDash$ SiP (and SeP $\vDash$ SoP). But in QL, $\forall v(\phi v \supset \psi v)$ does not logically imply $\exists v(\phi v \,\&\, \psi v)$. (a) *Consequence*. Some valid syllogisms (and valid conversions, see Handout 9) are invalid in QL, *viz*. those with universal premises and existential conclusions, such as Darapti (MaP, MaS $\vdash$ SiP): $\forall x(Mx \supset Px)$, $\forall x(Mx \supset Sx) \nvdash \exists x(Sx \,\&\, Px)$. (b) *Reason*: *existential import*: $(\exists v)$ implies that there exists what is designated by the variable. So, if the domain denoted by $v$ is *empty*, $\exists v(\phi v \,\&\, \psi v)$ is false. In contrast, $(\forall v)$ lacks existential import: even if the domain of $v$ is empty, the conditional $\forall v(\phi v \supset \psi v)$ is true, since conditionals with a false antecedent are true (cf. Handout 4). In QL, 'All unicorns are beautiful' is true *because* there are no unicorns. So, QL requires non-empty universes of discourse.

*The semantics of QL.*
In PL, the truth of a complex proposition is a function of the truth of the atomic propositions and their connectives. Evaluating propositions is to assign truth values (T/F) to them. In QL, assigning truth values is more complicated. The evaluation of truth depends on the semantic values of constants, sentences, predicates, and also on the universe of discourse (i.e. the domain of quantification):

(1) Domain **D** of discourse: non-empty set of objects;
(2) Semantic value of a constant: the individual to which it refers;
(3) Value of a predicate: the set of objects in **D**. For multiadic predicates: ordered tuples (e.g., $\{\langle a, b\rangle, \langle a, c\rangle, \ldots\}$.

| Objects in: | $Fx$ | $Gx$ | $Fx \,\&\, Gx$ | $Fx \vee Gx$ | $Fx \supset Gx$ |
|---|---|---|---|---|---|
| (a) | T | T | T | T | T |
| (b) | T | F | F | T | F |
| (c) | F | T | F | T | T |
| (d) | F | F | F | F | T |




(4) Note: variables have no semantic values. $Fx$ is like saying, e.g., 'she smiles'. This is the relevance of *binding variables*. Open wffs with free variables are merely propositional functions, rather than propositions that can be evaluated. Only wffs with all variables bound are sentences or propositions. To determine the truth of an open formula ($Fx \,\&\, Gy$), the formula must hence be closed by replacing variables with constants ($Fa \,\&\, Gb$), or by binding the free variables to a quantifier, e.g., $\exists x \forall y(Fx \,\&\, Gy)$.
(5) *Truth* for sentences with constants: $Fa$ is true iff the individual designated by 'a' has the property, or satisfies the predicate, specified by 'F'. For instance, 'Plato has a beard' is true iff Plato has a beard. This account of truth is *disquotational*. With this, we leave formal logic behind and enter the philosophical debate about truth.

(6) *Truth* for existential propositions: ∃*xFx* is true iff *Fa* is true: an instance *verifies* the existential proposition. Recall Ockham's 'descent' (see Handout 10): ∃*xFx* is equivalent to *Fa* ∨ *Fb* ∨ *Fc* ∨ …; existential propositions are true iff there is *at least one* variable assignment that satisfies the predicate, i.e. when the variable is taken to stand for an object in the predicate's extension. Likewise for more than one variable. (A sentence with subformulae requires the evaluation of its components, whose variables may thus become free and hence lack a truth value. In this case, variables are assigned temporary and arbitrary references.)

(7) ∀*xFx* is false iff ~*Fa* is true (i.e. iff *Fa* is false): a counterexample *falsifies* the universal proposition. Ockham's 'descent': ∀*xFx* is equivalent to *Fa* & *Fb* & *Fc* & …; for all individuals in **D**. Universal propositions are true iff *all* variable assignments satisfy the predicate. To decide whether ∀*x*(*Fx* ⊃ *Gx*) is true, say, we must know **D** and then we check whether all the individuals in **D** belong to the extensions of *F* and *G*, i.e. we evaluate *Fa* ⊃ *Ga*, *Fb* ⊃ *Gb*, *Fc* ⊃ *Gc*, …; working with a *truth table* is entirely impractical.

(8) Furthermore, since not all objects in **D** may have a name, this conjunctive strategy does not always work. Rather than as a name, we can think of the variable *v* as a *pronoun* that picks out any object in **D**, but no specific object. When we thus say, e.g., 'Everybody is smart', it does not matter which person in **D** we pick out. In a way, we select an arbitrary yet exemplary object.

*Validity in QL.*

Remember: an argument is valid iff the conclusion cannot be false *if* the premises are true. In PL, valid arguments are *tautologies*; in QL, valid arguments are *logical truths*. The validity of QL arguments can be tested either by natural deduction (as for PL but with additional rules, see below), or with so-called 'trees' (cf. Handout 14).

*Natural Deduction in QL Sketched.*

Generalisation: introducing quantifiers; instantiation: removing quantifiers.
(1) Universal Introduction (∀I): *ϕa* ⊢ ∀*vϕv*, but only if *ϕa* is neither a premise nor depends on any premises in which *a* (i.e. an arbitrary constant) occurs.
(2) Existential Elimination (∃E): ∃*vϕv* ⊢ *ϕa*, the move from something to a specific (typical) object is similar to ∨E, where the two disjuncts must lead to the same conclusion before the disjunction can be eliminated (cf. Handout 7). The ∀I and ∃E rules are rather unintuitive.
(3) Universal Elimination (∀E): ∀*vϕv* ⊢ *ϕa*, where *a* is an *arbitrary name* that replaces all occurrences of *v*.
(4) Existential Introduction (∃I): *ϕa* ⊢ ∃*vϕv*.

*Example*: ∀*x*(*Fx* ⊃ *Gx*), *Fa*, ⊢ ∃*x*(*Gx*)

| | | | |
|---|---|---|---|
| 1 | (1) | ∀*x*(*Fx* ⊃ *Gx*) | Premise |
| 2 | (2) | *Fa* | Premise |
| 1 | (3) | *Fa* ⊃ *Ga* | 2, ∀E |
| 1, 2 | (4) | *Ga* | 2, 3, ⊃E |
| 1, 2 | (5) | ∃*xGx* | 4, ∃I |