Formal Fallacies

A formal fallacy is an invalid argument grounded in logical form. *Validity*, again: an argument is valid iff the premises could not all be true yet the conclusion false; or, a rule of inference is valid iff it cannot lead from truth to falsehood, i.e. if the inference preserves truth. The semantic turnstile '⊨' indicates validity, so where **P** is the set of premises and C is the conclusion, **P** ⊨ C. We can also say that C is a 'logical consequence' of, or follows from, **P**. In a fallacious argument, this may *seem* to be the case, but **P** ⊭ C.

There are also many *informal* fallacies (e.g., begging the question: *petitio principii*) that do not preserve truth either, but these are *not* grounded in logical form.

Fallacies of Propositional Logic (PL)

(1) Affirming the Consequent: $\phi \supset \psi, \psi \vdash \phi$
Examples: (a) 'If Hume is an atheist, then Spinoza is an atheist too. Spinoza is an atheist. So, Hume is an atheist.' (b) 'If it rains, the street is wet. The street is wet. So it rains.'

(2) Denying the Antecedent: $\phi \supset \psi, \sim\phi \vdash \sim\psi$
Examples: (a) 'If Locke is a rationalist, then Spinoza grinds lenses for a living. But Locke is not a rationalist. So, Spinoza does not grind lenses for a living.' (b) 'If it rains, the street it wet. It does not rain. So the street is not wet.' Affirming the consequent and denying the antecedent are quite common fallacies. They ignore that the material implication lacks an *intrinsic* connection between antecedent and consequent. This is unlike a *determinative* relation (see Handout 4, the paradoxes of the material implication).

(3) Commutation of Conditionals:[1] $\phi \supset \psi \vdash \psi \supset \phi$
Examples. (a) 'If thought experiments work, we can trust our philosophical intuitions. So, if we can trust our philosophical intuitions, thought experiments work.' (b) 'If it rains, the street is wet. So, if the street is wet, it rains'.

(4) Improper Transposition: $\phi \supset \psi \vdash \sim\phi \supset \sim\psi$
Examples. (a) 'If utilitarians are right, suicide is not permissible. So, if utilitarians are not right, suicide is permissible.' (b) 'If God does not exist, then we are deceived. So, if God does exist, we are not deceived.'

P	Q	$(P \supset Q)$	$\supset$	$(\sim P \supset \sim Q)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

(5) Improper Disjunctive Syllogism (Affirming One Disjunct): $\phi \vee \psi, \phi \vdash \sim\psi$
Examples. (a) 'Either Hume or Berkeley is an empiricist. Hume is an empiricist. So, Berkeley is not.' (b) 'Romeo loves Juliet or Juliet loves Romeo. Juliet loves Romeo. So, Romeo does not love Juliet.'

1 The following names are from Names from Wilson, W. K. (1995). Formal Fallacy, in R. Audi (Ed.), *The Cambridge Dictionary of Philosophy* (pp. 272–3). Cambridge: Cambridge University Press.

Fallacies of Syllogistics

(6) Four Terms, *quaternio terminorum*: due to ambiguous middle term.
Examples. (a) 'Nothing is better than eternal bliss. A glass of water is better than nothing. So, a glass of water is better than eternal bliss.' (b) 'All moral philosophers like thought experiments. Some philosophers cycle. So, some cyclists like thought experiments.'

(7) Undistributed Middle Term (for distribution, see Handouts 9 and 10).
Example. 'Some rationalists dance. Some rationalists do not cycle. So, Some cyclists dance.'

(8) Illicit Major (Predicate term distributed in conclusion but not in major premise)
Example. 'All smart people are philosophers. No tweeter is smart. So, some tweeters are not philosophers.'

(9) Illicit Minor (Subject term distributed in conclusion but not in minor premise)
Example. 'All cyclists are philosophers. All cyclists live dangerously. So, all philosophers live dangerously.'

(10) Illicit Affirmative: E/O, E/O ⊢ A/I
Example. 'No philosopher smokes. No smoker likes logic. So, all philosophers like logic.'

(11) Illicit Negative: A/I, A/I ⊢ E/O
Example. 'All cyclists like Descartes. Whoever likes Descartes hates Locke. So, no cyclist hates Locke.'

Fallacies of Predicate Logic (QL, QL=)

(12) Illicit Quantifier Shift: $\forall x \exists y Rxy \vdash \exists y \forall x Rxy$
Example. 'Everybody loves somebody. So, somebody loves everybody.'

(13) Unwarranted Contrast: $\exists x(Fx \,\&\, Gx) \vdash \exists x(Fx \,\&\, \sim Gx)$
Example. 'Some politicians are honest. So, some politicians are not honest.'
Compare: SiP ⊢ SoP. SiP and SoP are subcontraries (cf. Handout 9, the Square of Opposition), hence both S and P may be true but one must be true (i.e. S and P cannot both be false).

(14) Illicit Substitution of Identicals ('Masked Man'): $\mathsf{B}Fa$, $a = b \vdash \mathsf{B}Fb$, where B is the modal belief operator.
Example. 'Jim believes that George Orwell wrote *1984*. But George Orwell is Eric Blair. So, Jim believes that Eric Blair wrote *1984*.'

Fallacies of Modal Logic

(15) Modal Fallacy: $\Box(\phi \supset \psi) \vdash \phi \supset \Box\psi$
Example. '[a] It is necessary that if Adam exists, then he accepts the apple. So, [b] if Adam exists, he necessarily accepts the apple.' (cf. Leibniz, *Discourse* §13). While [a] expresses the necessity of the *consequence*, [b] expresses the necessity of the *consequent*. This is similar to the *de dicto* and a *de re* reading, respectively (cf. Handout 15). Leibniz calls the latter a 'hypothetical necessity', because Adam's sinning is necessary only on the condition or hypothesis that he exists. (Leibniz does not commit the fallacy, of course.)