Non-Classical Logic

Propositional Logic, Syllogistics, and Predicate Logic are the *sediment* of symbolic logic, which has slowly formed since Aristotle's first systematic accounts in the 'Organon'.[1] But philosophical logic is still a work in progress. Here are two ideas that have emerged during the last eighty years or so.

Many-Valued Logic, MVL (Development of Propositional Logic)
Lukasiewicz writes (1930): "I can assume without contradiction that my presence in Warsaw at a certain moment in time next year, e.g., at noon on 21st December, is not settled at the present moment either positively or negatively. It is therefore *possible but not necessary* that I shall be present in Warsaw at the stated time. On this presupposition the statement 'I shall be present in Warsaw at noon on 21st December next year' is neither true nor false at the present moment. For if it were true at the present moment my future presence in Warsaw would have to be necessary, which contradicts the presupposition, and if it were false at the present moment, my future presence in Warsaw would have to be impossible, which again contradicts the presupposition. The statement under consideration is therefore at the present neither true nor false and must have a third value different from 0, or the false, and 1, or the true. We can indicate this by '½': it is 'the possible', which goes as a third value with 'the false' and 'the true'. This is the train of thought which gave rise to the three-valued systems of propositional logic."[2]

Compare Aristotle's 'sea-battle' in *De Interpretatione* 9 (18b10ff.): *bivalence* demands that either of these contradictories is true: (a) 'There will be a sea-battle tomorrow', (b) 'There will not be a sea-battle tomorrow'. But if (a) is true *now*, tomorrow's battle is necessary, and if (b) is true, the battle is impossible. There is no room to say that it *might* happen. Hence, for Aristotle, 'it is not necessary that of every affirmation and opposite negation one should be true and the other false' (*op. cit.*, 19b1–2).

Features of MVL

(1) Rejects the semantic principle of bivalence; allows for three or more truth values: T, F, and ½ or U (undetermined); but a truth value t could be $0 \leq t \leq 1$. Here are Lukasiewicz's truth tables:

ϕ	$\sim\phi$
0	1
½	½
1	0

$\vee$	0	½	1
0	0	½	1
½	½	½	1
1	1	1	1

&	0	½	1
0	0	0	0
½	0	½	½
1	0	½	1

$\supset$	0	½	1
0	1	1	1
½	½	1	1
1	0	½	1

ϕ	$\Diamond\phi$
0	0
½	1
1	1

ϕ	$\Box\phi$
0	0
½	0
1	1

1 The 'Organon' is the set that comprises *Categories*, *De Interpretatione*, *Prior Analytics*, *Posterior Analytics*, *Topics*, *On Sophistical Refutations*.

2 In Kneale, W. & Kneale, M. (1962). *The Develoment of Logic* (pp. 569–70). Oxford: Clarendon Press.

(2) MVL rejects the Law of the Excluded Middle, i.e. $\phi \vee \sim\phi$.
(3) MVL rejects the Law of Non-Contradiction, i.e. $\sim(\phi \,\&\, \sim\phi)$.
(4) MVL can be applied to *vague* contexts, and is hence called 'fuzzy logic'. Take the propositions (a) 'Socrates is ugly', or (b) 'Spinoza's *Ethics* is difficult'. Here, we might say that the truth or falsity of these propositions 'depends', or is in some sense *indeterminate* or indefinite.

Free Logic, FL (Development of Predicate Logic)
One assumption of QL is that singular terms denote entities that exist. The universe of discourse **D** should not be empty. But some terms *are* empty, i.e. fail to refer to anything: 'Sherlock Holmes', 'Pegasus', 'Descartes's Seventh Meditation'. Classical QL does not distinguish between denoting and non-denoting constants: if we can use a term, it denotes. FL *allows* non-denoting terms, or terms that might denote.

Features of FL
(1) FL rejects existential import: it is free insofar as it is free of existential assumptions. There may be individual constant, free variables, and descriptions that fail to denote anything.
(2) FL thus rejects a series of classical assumptions and inferences: (a) the schema $\exists x\; x{=}a$ is not a logical truth; (b) existential generalisation ($\exists$I), and (c) universal instantiation ($\forall$E) are invalid (see Handout 13).
Classic logic: $\forall xFx \supset Ft$, where t is a singular term;
FL: $\forall xFx \supset$ (t exists $\supset Ft$), or $\forall xFx \supset \exists y(y = t) \supset Ft$.
(3) The main issue: settle truth conditions or a semantics for sentences with non-referring terms. In FL, 'Sherlock Holmes does not wear a hat' could be true. In classical logic, the proposition is false because the existence of Holmes is presumed (existential import). And if we say 'Sherlock Holmes does not exist' ($\sim\exists x\; x = \mathrm{s}$), we get a contradiction, for we assume the very thing that we claim does not exist: 'Non-being must in some sense be, otherwise what is it that there is not?'[3]
(4) The correspondence theory of truth does not work: 'Pegasus is white' is true iff *there is* Pegasus and it is white. An alternative semantics is *supervaluation*. Here is the rough idea.[4] If a complex proposition has non-denoting terms, a truth value can still be evaluated by considering the values that its components would have if the terms had referents. *Example*. Proposition N: $\sim(Ft \,\&\, \sim Ft)$, where t lacks reference. In a neutral semantics for FL, where non-denoting atomic formulas are truth-valueless, N lacks a truth value. But not on a supervaluational account: *if t was* in the extension of *F*, then N *would* be true, and *if t was not* in the extension of *F*, then N *would* be true too. So, N is 'supertrue' (and the law of non-contradiction is valid in FL).

3 Quine, W. V. (1948). On What There is (p. 21). *Review of Metaphysics*, *2*, 21–38.
4 See, e.g., Nolt, J. (2010). Free Logic (sec. 3.4), in E. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*, http://plato.stanford.edu/entries/logic-free/.