

Propositional Logic I

An argument is a pattern of inference. It is *valid* if there is no interpretation under which all the argument's premises are true but the conclusion is false. Arguments are valid if they do not produce falsehoods from true premises.

Propositions

A proposition is an entity that has a truth value. It is either true (T) or false (F).

Usually, a proposition is expressed in a sentence of a natural language:

- (1) 'Jim likes Wagner' is an English sentence that expresses the proposition *that* Jim likes Wagner; or
- (2) 'Descartes aimait le vin' is a French sentence that expresses the proposition *that* Descartes liked wine.

Preliminaries

- (a) Greek letters (ϕ and ψ) are metavariables, which stand for *any* sentence of the language of propositional logic (PL). They are metalinguistic variables that we use to think about the object language, which is here PL.¹ Occasionally, A , B , and C are used as metavariables.
- (b) There is a distinction between *use* and *mention*. Quotation marks allow us to mention a word or sentence: (i) Socrates has a beard; (ii)* 'Socrates' has a beard; (iii) 'Der Hund bellt' is a German sentence; (iv) The formula ' $P \vee Q$ ' is a sentence in PL; (v)* P means that Socrates is bearded.

Approaching PL

- (3) Jim plays the piano or Jackie sings. Jackie does not sing. So Jim plays the piano.
- (4) (Jim plays the piano) *or* (Jackie sings). *Not* (Jackie sings). *So* (Jim plays the piano).
- (5) Lexicon or interpretation: $P =$ (Jim plays the piano), $Q =$ (Jackie sings)
- (6) P or Q . *Not* Q . *So* P .
- (7) $P \vee Q, \sim Q \therefore P$, where ' \therefore ' is an inference marker that means 'therefore' in PL.²
This is sometimes also put in a vertical way, including a line as the inference marker:

$$\begin{array}{c} P \vee Q \\ \sim Q \\ \hline P \end{array}$$

The aim of PL is to represent or express how propositions are connected. Translation into PL can reveal the logical structure of natural languages.

¹ Similarly, ' \vDash ' is a metalinguistic symbol too; it does not belong to PL.

² No quotation marks in the object language; in metalanguage: ' $(P \vee Q)$ ', ' $\sim Q$ ' \vDash ' P ' says that the PL inference which concludes from ' $(P \vee Q)$ ' and ' $\sim Q$ ' to ' P ' is valid.

Syntax of PL

The grammar of PL defines which symbols are meaningful sentences in PL. The syntax sets out what counts as a well-formed formula (wff) in that language.

- (i) Basic sentence letters or atomic formulae: P, Q, R, S , etc. (with or without indices for more sentences)³
- (ii) Connectives or logical constants: $\&, \vee, \sim, \supset, \equiv$ (connectives generate complex formulae)
- (iii) Brackets for scope: $(,)$

Definition of a wff in PL:

- Any atomic formula is a wff.
- If ϕ and ψ are wff, then $(\phi \& \psi)$, $(\phi \vee \psi)$, $\sim\phi$, are also wffs.⁴
- Nothing else is a wff in PL.

Semantics of PL

PL needs an interpretation, which makes its sentences meaningful and available to a truth evaluation. (Remember: propositions have truth values.)

Connective	English	Symbol
Negation ⁵	‘it is not the case that’	\sim or \neg
Conjunction	‘and’, ‘both’	$\&$ or \wedge or \cdot
Disjunction	‘or’, ‘at least one’	\vee
Material Implication, Conditional	‘if ... then’	\supset or \rightarrow
Material Equivalence, Biconditional	‘if and only if’	\equiv or \leftrightarrow

Truth Functionality: the truth of a complex formula is a function of the truth of its constituent formulae. The connectives are defined by a specific distribution of truth values (T, F, or 1, 0). There are 16 such distributions for dyadic connectives.

ϕ	$\sim\phi$	ϕ	ψ	$\phi \& \psi$	$\&$	T	F	ϕ	ψ	$\phi \vee \psi$	\vee	T	F
T	F	T	T	T	T	T	F	T	T	T	T	T	T
F	T	T	F	F	F	F	F	T	F	T	F	T	F
		F	T	F				F	T	T			
		F	F	F				F	F	F			

The *conjunction* of ϕ and ψ is true if and only if ϕ and ψ are both true, and false otherwise. The (inclusive) *disjunction* of ϕ and ψ is true if and only if at least one of ϕ and ψ is true, and false otherwise.

3 In older English and current German texts, small letters p, q , etc. are often used.

4 The sentences $(\phi \supset \psi)$ and $(\phi \equiv \psi)$ are also wffs. But they can be reduced to those mentioned above. Note too that more precisely, we should say: if ϕ and ψ are wff, so is $\ulcorner (\phi \& \psi) \urcorner$. The new quotation marks (‘Quine corners’) make clearer that the symbols of the object language are mentioned while the metasympols are used. So, $\ulcorner (\phi \& \psi) \urcorner$ means “the result of writing ‘(’ followed by ϕ (any atomic formula of PL) followed by ‘&’ followed by ψ followed by ‘)’.”

5 The negation is a *monadic* ‘connective’, which only takes one wff. The other connectives are dyadic, and take two wffs.

