Propositional Logic II

It is ambiguous to talk *about* the logical structure of a natural language *in* that very language. The formal language PL avoids confusion. But the same holds for PL itself: we should not talk about PL in terms of PL. Hence we also need a metalanguage. (And so on.)

The logical form of natural sentences often needs to be teased out: 'Unless Sue speaks to me and confirms the date, I cannot arrange to see Jack and this is is not good', turns into, '(((Sue speaks to me) and (Sue confirms the date)) or (it is not the case that (I arrange to see Jack) and it is not the case that (this is good)))', and further into, '$((P \& Q) \vee (\sim R \& \sim S))$'.

Further Connectives

ϕ	ψ	$\phi \vee \psi$
T	T	T
T	F	T
F	T	T
F	F	F

ϕ	ψ	$\phi >\!\!-\!\!< \psi$
T	T	F
T	F	T
F	T	T
F	F	F

ϕ	ψ	$\phi \mid \psi$
T	T	F
T	F	T
F	T	T
F	F	T

(1) The (inclusive) *disjunction* '$\vee$' of ϕ and ψ is true iff at least one of ϕ and ψ is true, and false otherwise (one or the other or both but not neither; at least one).

(2) The *contravalence* '>–<'(or 'xor') is true iff either ϕ or ψ is true and false when none or both ϕ and ψ are true (one or the other but neither both nor none).

(3) The e*xclusion* '|' (Sheffer stroke) is true iff either ϕ or ψ is true, or if neither ϕ nor ψ is true, and false if both ϕ and ψ are true (one or the other or neither but not both; at most one).

(4) The *material implication* (conditional) '$\supset$' is true iff ϕ is false or ψ is true (not ϕ without ψ). The conditional '$\phi \supset \psi$' is *false* iff the *antecedent* ϕ is true and the *consequent* ψ is false. This is central to understanding *validity*.

ϕ	ψ	$\phi \supset \psi$
T	T	T
T	F	F
F	T	T
F	F	T

ϕ	ψ	$\phi \equiv \psi$
T	T	T
T	F	F
F	T	F
F	F	T

(5) The *equivalence* is true iff ϕ and ψ have the same truth values (both or none).

(Some) Reductions

$\phi >\!\!-\!\!< \psi \equiv \sim(\phi \equiv \psi)$ (contravalence is equivalent to the negation of equivalence)
$\phi \mid \psi \equiv \sim(\phi \& \psi)$ (exclusion is equivalent to the negation of the conjunction)
$\phi \urcorner \psi \equiv \sim\phi$ (prenonpendence is equivalent to the negation of ϕ)
$\phi \dagger \psi \equiv \sim(\phi \vee \psi)$ (rejection is equivalent to the negation of the disjunction)
$\phi \supset \psi \equiv (\sim\phi \vee \psi)$ (cf. definition)
$\phi \supset \psi \equiv \sim(\phi \& \sim\psi)$

There is no need for sixteen separate dyadic connectives. The small stock of '~', '&', '$\vee$', and '$\supset$' is enough.

All Dyadic Operators

ϕ	ψ	*Tautology* $\phi \top \psi$	*Disjunction* $\phi \vee \psi$	*Replication* $\phi \subset \psi$	*Implication* $\phi \supset \psi$	*Exclusion* $\phi \mid \psi$	*Equivalence* $\phi \equiv \psi$	*Prenon-pendence* $\phi \rceil \psi$	*Postnon-pendence* $\phi \lceil \psi$
T	T	T	T	T	T	F	T	F	F
T	F	T	T	T	F	T	F	F	T
F	T	T	T	F	T	T	F	T	F
F	F	T	F	T	T	T	T	T	T

ϕ	ψ	*Post-pendence* $\phi \lfloor \psi$	*Pre-pendence* $\phi \rfloor \psi$	*Contra-valence* $\phi >\!\!-\!\!< \psi$	*Conjunction* $\phi \,\&\, \psi$	*Postsection* $\phi >\!\!- \psi$	*Presection* $\phi -\!\!< \psi$	*Rejection* $\phi \dagger \psi$	*Contra-diction* $\phi \perp \psi$
T	T	T	T	F	T	F	F	F	F
T	F	F	T	T	F	T	F	F	F
F	T	T	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F	T	F

Conditionals for Stoics

'[A] conditional either begins from a truth or ends in a truth (e.g., If it is day, it is light), or begins from a falsity and ends in a falsity (e.g., If the earth is flying, the earth has wings), or begins from a truth and ends in a falsity (e.g., If the earth exists, the earth is flying), or begins in a falsity and ends in a truth (e.g., If the earth is flying, the earth exists). Of these they say that only those beginning from a truth and ending in a falsity are unsound and that the others are sound.'[1]

Paradoxes (violations of intuitions) of the Implication

(1) If the *antecedent* is false, the conditional is true (ϕ : F, $\phi \supset \psi$: T). Hence, *ex falso sequitur quodlibet*, i.e. from a falsehood anything follows.

(2) Whenever the *consequent* is true, the whole conditional is true (ψ : T, $\phi \supset \psi$: T). Hence, *verum sequitur ex quodlibet*, i.e. truth follows from anything.

This yields two *valid* arguments.

(3) '$\sim\phi$' $\vDash$ '$\phi \supset \psi$'. So, if 'dogs do not bark' is true, i.e. when 'dogs bark' is false, then 'if dogs bark then swans are white' is true too.

(4) 'ψ' $\vDash$ '$\phi \supset \psi$'. So, if 'grass is green' is true, then 'if dogs are furry then grass is green' is true too. In *natural deduction*, this is known as '$\supset$Intro'.

This is the price for the *truth functionality* of the material impliction: an connective is truth functional if it is possible to determine the truth-value of a complex proposition solely on the basis of the truth-values of its components. The *content* of P and Q is irrelevant for the truth of $P \supset Q$. So, P and Q are *not really connected*. But this connection *is* a requirement for ordinary or quotidian 'if…then' statements. The paradoxes arise because the mutual relevance between P and Q is lost.

1 Sextus Empiricus, *Outlines of Pyrrhonism* II.105. Transl. J. Annas & J. Barnes (2000), Cambridge University Press (p. 94).