How to Evaluate Complex PL Sentences

Consider the complex proposition (~(*P* ⊃ *Q*) ∨ (*P* ∨ (~*Q* & ~*P*))). It contains two atomic propositions or sentence letters (*P*, *Q*). Hence there are $2^2 = 4$ possible situations to evalute or calculate. But let us start with one, where both *P* and *Q* are true (have truth value T).

First, identify the main connective, which here is '∨' (see the arrow). This is where the final truth value for the entire proposition will eventually stand. It is here underlined for clarity. But before we can say whether the entire proposition is false given tht *P* and *Q* are true, we have to work our way upwards or outwards from the atomic propositions. Remember: the truth value of a complex proposition is a function of the truth values of its constituent atomic propositions. The numbers suggest a possible order. (See explanatory text below.)

						↓							
P	Q	~	(P	⊃	Q)	∨	(P	∨	(~	Q	&	~	P))
T	T	F	T	T	T	T	T	T	F	T	F	F	T
		6	1	5	1	7	1	4	2	1	3	2	1

Begin by repeating the truth values under each letter as they are in the first two columns (1). Then, tackle the second part of the proposition, beginning with the negations (2). After this, the innermost bracket with the conjunction can be evaluated (3), and subsequently the disjunction (4). Now turn your attention to the first part of the proposition and evaluate the implication (5). Next, the negation of the implication (6). Finally, evaluate the main connective (disjunction) (7): since the first part is false and the second part is true, the entire proposition is true.

So far, we only know that the proposition is true under the condition that *P* and *Q* are both true. Hence there are three more cases to consider. Deal with them as above.

P	Q	~	(*P*	⊃	Q)	∨	(*P*	∨	(~	Q	&	~	*P*))
T	F	T	T	F	F	T	T	T	T	F	F	F	T
		6	1	5	1	7	1	4	2	1	3	2	1

Now we know that the proposition is true given that *P* is true and Q is false.

P	Q	~	(*P*	⊃	Q)	∨	(*P*	∨	(~	Q	&	~	*P*))
F	T	F	F	T	T	F	F	F	F	T	F	T	F
		6	1	5	1	7	1	4	2	1	3	2	1

Now we know that the proposition is false when *P* is false but *Q* is true. One more situation to evaluate

P	*Q*	~	(*P*	⊃	*Q*)	∨	(*P*	∨	(~	*Q*	&	~	*P*))
F	F	F	F	T	F	T	F	T	T	F	T	T	F
		6	1	5	1	7	1	4	2	1	3	2	1

Of course, if you want to evaluate *all* possible situations, it makes more sense to start right away with a full truth table, as shown next. The procedure however remains the same.

P	*Q*	~	(*P*	⊃	*Q*)	∨	(*P*	∨	(~	*Q*	&	~	*P*))
T	T	F	T	T	T	T	T	T	F	T	F	F	T
T	F	T	T	F	F	T	T	T	T	F	F	F	T
F	T	F	F	T	T	F	F	F	F	T	F	T	F
F	F	F	F	T	F	T	F	T	T	F	T	T	F

Now, consider this proposition: (~(*P* & ~(~*Q* ∨ *R*))). It contains three atomic propositions, which yields $2^3 = 8$ possible situations to evaluate (calculate). Hence, the full truth table looks like this:

P	Q	R	~ (	P	&	~ (	~	Q	∨	R))
T	T	T	T	T	F	F	F	T	T	T
T	T	F	F	T	T	T	F	T	F	F
T	F	T	T	T	F	F	T	F	T	T
T	F	F	T	T	F	F	T	F	T	F
F	T	T	T	F	F	F	F	T	T	T
F	T	F	T	F	F	T	F	T	F	F
F	F	T	T	F	F	F	T	F	T	T
F	F	F	T	F	F	F	T	F	T	F

With practice, you will begin to skip some steps. For instance, it is not always necessary to list all initial truth values once again. A more relaxed table can then look like this final one. But all connectives should get full evaluations.

P	Q	R	~ (	P	&	~ (	~	Q	∨	R))
T	T	T	T		F	F	F		T	
T	T	F	F		T	T	F		F	
T	F	T	T		F	F	T		T	
T	F	F	T		F	F	T		T	
F	T	T	T		F	F	F		T	
F	T	F	T		F	T	F		F	
F	F	T	T		F	F	T		T	
F	F	F	T		F	F	T		T	