Syllogistics I

Aristotle is among the first to explore the *validity* of argment-*forms*, i.e. schemata of arguments whose premises provide conclusive grounds for the truth of the conclusion (cf. Handout 1):
"If *M* belongs to [ὑπάρχειν, also to predicate, or apply to] every *N* but [and] to no *X*, then neither will *N* belong to any *X*. For if *M* belongs to no *X*, then neither does *X* belong to any *M*; but [and] *M* belonged to every *N*; so, *X* will belong to no *N* […]. And since the privative [negative] converts, *N* will belong to any *X* […]." (*Prior Analytics* 27a9–13)[1]

Among others (e.g., the Stoics, Boethius, Aquinas, Abelard) *Ockham* (c. 1280–1349) develops *The Philosopher's* logic into something we recognise today.[2]

(a) "For the truth of a copulative proposition, it is required that each part be true (*Ad veritatem copulativae requiritur, quod utraque pars sit vera*)", which is the truth-functional definition of '&'.

(b) "It must be understood that from either part of a disjunctive proposition to the entire disjunctive proposition there is a valid argument (*quod ab altera parte disjunctivae ad totam disjunctivam est bonum argumentum*)", which is ∨I of natural deduction.

(c) "It must also be known that there is a valid inference from a copulative proposition to each part of it (*quod semper a copulativa ad utraquem partem est consequentia bona*)", which is &E.

Ockham also knew 'De Morgan's Laws': (1) "We should also know that the contradictory opposite of a copulative proposition is a disjunctive proposition composed of the contradictory opposites of its parts", and (2) "It must be understood that the contradictory opposite of a disjunctive proposition is a copulative proposition composed of the parts of the disjunctive proposition". Translated into PL:

$$(1)\ {\sim}(P \,\&\, Q) \equiv ({\sim}P \vee {\sim}Q) \qquad (2)\ {\sim}(P \vee Q) \equiv ({\sim}P \,\&\, {\sim}Q)$$

Since propositional logic deals with complete statements, and hence fails to express complex arguments like this:

Socrates is human	*P*
All humans are mortal	*Q*
Therefore, Socrates is mortal	∴ *R*

The argument looks valid (and sound), yet '*P*', '*Q*' ⊢ '*R*' is not a valid PL sequent. Syllogistics (and Predicate Logic) can accommodate this. Instead of whole sentences, syllogistics deals with *terms* in sentences.

1 For a detailed overview, see http://plato.stanford.edu/entries/aristotle-logic/.
2 See Boehner, P. & Brown, S. (1990). *Ockham: Philosophical Writings*. Indianapolis: Hackett.

Definition. A syllogism (συλλογισμός) is a valid sequence of three categorical (κατήγορειν, to predicate) propositions with three terms.

The *first* proposition is the major premise (πρότασις). It contains the major and the middle terms. The major term is the *predicate* (P) of the conclusion.

The *second* proposition is the minor premise, which contains the minor and the middle terms. The minor term is the *subject* (S) of the conclusion.

The *third* proposition is the conclusion (συμπέρασμα). It contains the major and minor terms.

The *middle* term (M) appears once in each premise, but not in the conclusion.

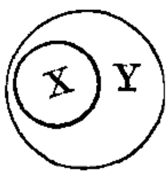
Categorical Propositions make assertions about *classes*, class membership, and mutual inclusion or exclusions. They conform to the schema: quantifier (subject term) copula (predicate term). There are four types of categorical propositions:

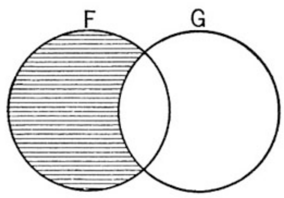
Quality → ↓ Quantity	Affirmative (*affirmo*)	Negative (*nego*)
Universal	*All A are B* AaB	*No A is B* AeB
Particular	*Some A are B* AiB	*Some A are not B* AoB

Examples

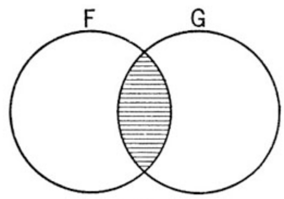
All Cretans are liars.
No Cretans are liars.
Some Cretans are liars.
Some Cretans are not liars.

Categorial statements can be expressed in Venn Diagrams.[3]

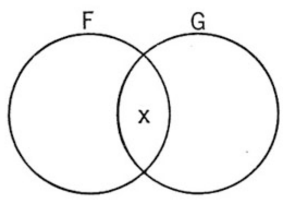




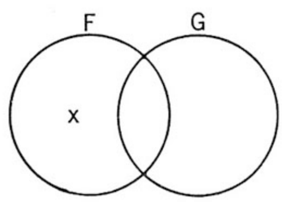
A: All *F* are *G*
DIAGRAM 1



E: No *F* are *G*
DIAGRAM 2



I: Some *F* are *G*
DIAGRAM 3



O: Some *F* are not *G*
DIAGRAM 4

3 Venn, J. (1880). On the Diagrammatic and Mechanical Representation of Propositions and Reasonings. *Philosophical Magazine*, *Series 5*, *Vol. 9*, *No. 59*, 1–18. Quine, W. V. O. (1962). *Methods of Logic*. London: Routledge & Keegan Paul (p. 69).