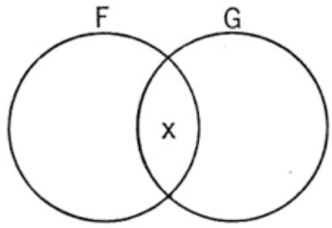
# More on Reading QL

What is the difference between these four QL sentences?

| | |
|---|---|
| (1) $\exists x(Fx \,\&\, Rxa)$ | *Lexicon*: |
| (2) $\exists x(Fx \supset Rxa)$ | $F$ = being a philosopher; $R$ = … knows …; |
| (3) $\forall x(Fx \supset Rxa)$ | $a$ = the 'Critique of Pure Reason' (CPR); |
| (4) $\forall x(Fx \,\&\, Rxa)$ | the domain **D** = persons. |

(1) says: there is at least one person *x* who is a philosopher and *x* knows the CPR. We could also say: at least one philosopher *x* knows the CPR; or, there is a philosopher *x* who knows the CPR.

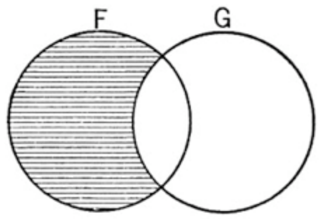
This is equivalent to a SiP categorical statement (cf. diagram).[1] Why? The SiP proposition says that at least one *F* is such that it also is *G*. And this is what (1) says.



**I**: Some *F* are *G*

Instead of a conjunction, (2) has a material implication as main connective in the scope of the existential quantifier. It may seem that (2) expresses the same thought as (1). But it does not. (2) reads: there is at least someone *x* such that if *x* is a philosopher then *x* knows the CPR. We could also say: there is someone *x* who, if *x* is a philosopher, *x* knows the CPR. Or: there is someone *x* who is a philosopher only if *x* knows the CPR. The point is that (2) is *weaker* than (1). Why? Think of the truth tables for '&' and '⊃'. (1) is true just in case there is an *x* that it is *both* a philosopher and knows the CPR. But (2) is made true by anyone who is *not* a philosopher. (A conditional with a false antecedent is true.)

(3) expresses this thought: every person *x* is such that if *x* is a philosopher, *x* knows the CPR. Or: everyone *x* who is a philosopher knows the CPR. Or, quite simply: all philosophers know the CPR. This is equivalent to a SaP categorical statement. Why? The SaP proposition says that all the *F* are *G*, so that there is no *F* that is not also *G*. In short: *if* there is an *F*, it is also *G* (cf. diagram). This is what (3) says.



**A**: All *F* are *G*

What about (4)? It says: everyone *x* is such that *x* is a philosopher and *x* knows the CPR. In other words: everybody is a philosopher who knows the CPR. This is obviously stronger than (3).

1 Quine, W. V. O. (1962). *Methods of Logic*. London: Routledge & Keegan Paul (p. 69).

In *Notes and Exercises* (p. 41), there are some hints at how the quantifiers relate. Here are some further details.

(5) $\exists x Rxa$,

which says that there is at least someone who knows the CPR. So, we can say that it is not the case that nobody knows the CPR. The proposition that nobody knows the CPR can be expressed with a universal quantifier; roughly, everybody is such that they do *not* know the CPR:

(6) $\forall x{\sim}Rxa$

Now, we can recapture (5) by using (6): it is not the case that for all persons *x* it is the case that they do not know the CPR:

(7) ${\sim}\forall x{\sim}Rxa$; or: ${\sim}(\forall x){\sim}Rxa$.

So, (5) and (7) are equivalent. What about (8)?

(8) $\forall x Rxa$,

which says that everybody knows the CPR. This is equivalent to saying that there is nobody who does not know the CPR. We can express this using the existential quantifier. First, we can say that nobody knows the CPR like this:

(9) ${\sim}\exists x Rxa$,

which says that it is not the case that there is someone who knows the CPR. Now, we can move to (10), and add a further negation, *viz*. that there is not anyone who does *not* know the CPR:

(10) ${\sim}\exists x{\sim}Rxa$, or: ${\sim}(\exists x){\sim}Rxa$.

So, (8) and (10) are equivalent. Furthermore, to say that not everybody knows the CPR is equivalent to saying that there are some who do not know it. Hence:

(11) ${\sim}\forall x Rxa \equiv \exists x{\sim}Rxa$.

Finally, to say that everybody does not know the CPR, or, more elegantly, that nobody knows the CPR (see (6) above), is equivalent to saying that there is not anyone who knows the CPR, which is what (9) says. So (6) and (9) are equivalent too:

(12) $\forall x{\sim}Rxa \equiv {\sim}\exists x Rxa$.