SOLUTIONS

Second Semantic Method: Trees

1.(a) {~*P* ∨ *Q*, *P* & ~*Q*}

(1)	~*P* ∨ *Q*		√	Assumption
(2)	*P* & ~*Q*		√	Assumption
(3)	*P*			2, &A
(4)	~*Q*			2, &A
(5)	~*P* *	*Q* *		1, ∨A

Both branches 'close' (*), which means we found inconsistencies.

1.(b) {*P*, *P* & *Q*, *P* ∨ ~*Q*}

(1)	*P*			Assumption
(2)	*P* & *Q*		√	Assumption
(3)	*P* ∨ ~*Q*		√	Assumption
(4)	*P*			2, &A
(5)	*Q*			2, &A
(6)	*P* ○	~*Q* *		3, ∨A

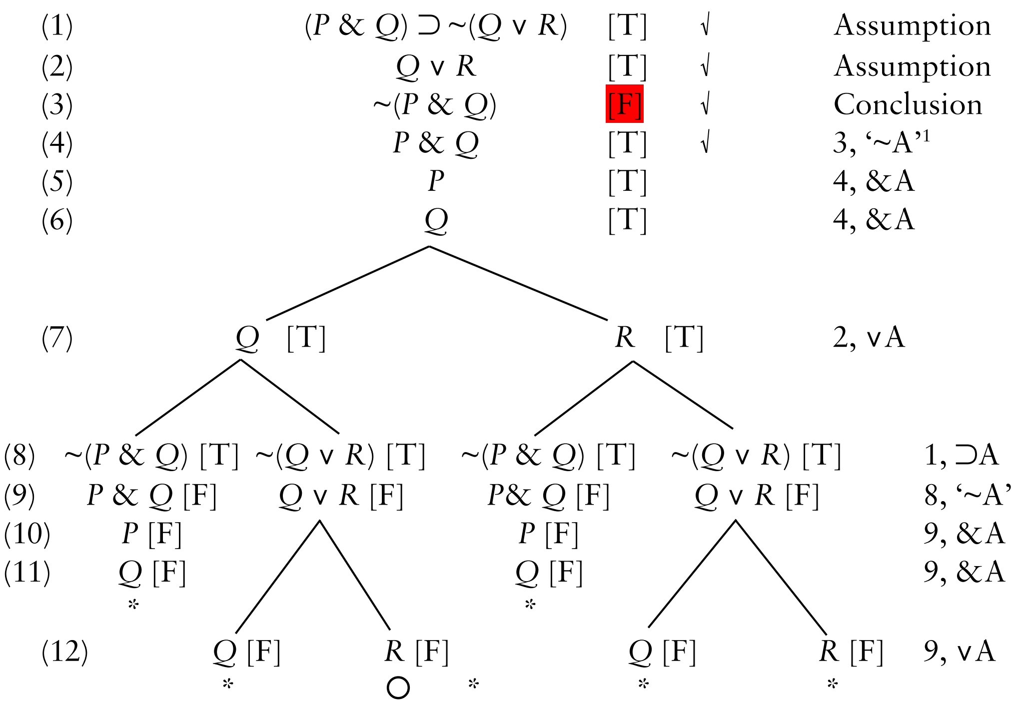
Only one branch closes, the other is open: the propositions are consistent.

1.(c) {*P* ∨ *Q*, ~*P* ⊃ ~*Q*, ~*P* & *Q*}

(1)	*P* ∨ *Q*			√	Assumption
(2)	~*P* ⊃ ~*Q*			√	Assumption
(3)	~*P* & *Q*			√	Assumption
(4)	~*P*				3, &A
(5)	*Q*				3, &A
(7)	*P* *	*Q*			1, ∨A
(8)		*P* *	~*Q* *		2, ⊃A

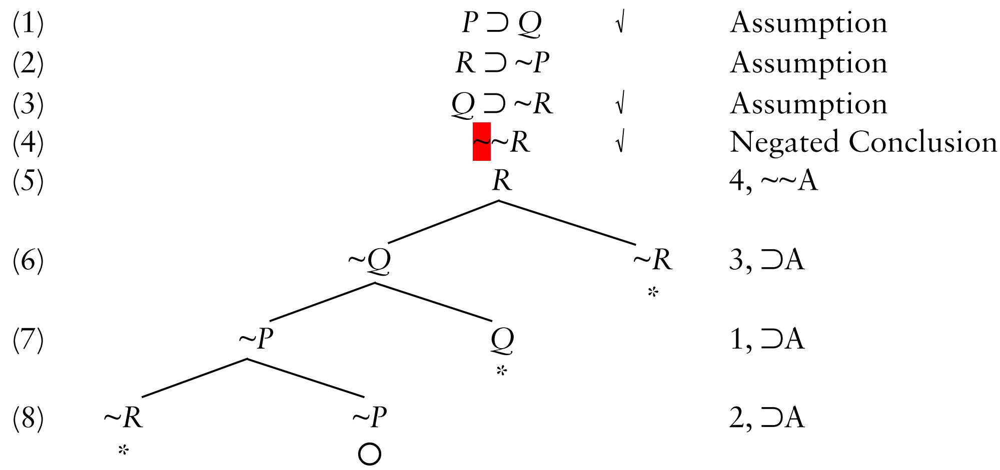
All branches close: the propositions are inconsistent.

2.(c) ‘(*P* & *Q*) ⊃ ~(*Q* ∨ *R*)’, ‘*Q* ∨ *R*’ ⊧ ‘~(*P* & *Q*)’



Since there is one open branch, the argument is invalid. Why? Because we find that the set of true premises and the false conclusion is *consistent*. If the argument is valid, this should not be the case. So, it is invalid.

3.(b) ‘*P* ⊃ *Q*’, ‘*R* ⊃ ~*P*’, ‘*Q* ⊃ ~*R*’ ⊧ ‘~*R*’



There is one open branch, which indicates a consistency. So the argument is invalid.

1 Removing negations like this works only with *signed* trees (cp. the rules on p. 29)!

3.(c) ‘(*P* ∨ *Q*) ⊃ *R*’ ⊧ ‘*P* ⊃ ~*R*’

(1)	(*P* ∨ *Q*) ⊃ *R*		√	Assumption
(2)	~(*P* ⊃ ~*R*)		√	Negated Conclusion
(3)	*P*			2, ~⊃A
(4)	~~*R*			2, ~⊃A
(5)	*R*			4, ~~A
(6)	~(*P* ∨ *Q*)	*R*		1, ⊃A
(7)	~*P*	O		6, ~∨A
(8)	~*Q*			6, ~∨A
	O			

All branches are open: the argument is invalid.

3.(d) ‘*P* ⊃ *Q*’ ⊧ ‘*P* & ~*Q*’

(1)	*P* ⊃ *Q*				√	Assumption
(2)	~(*P* & ~*Q*)				√	Negated Conclusion
(3)	~*P*		*Q*			1, ⊃A
(4)	~*P*	~~*Q*	~*P*	~~*Q*		2, ⊃A
(5)	O	*Q*	O	*Q*		4, ~~A
		O		O		

Not a single branch closes: the set is maximally consistent. Hence the argument is invalid.

3.(e) ‘*P* ⊃ *Q*’, ‘~(*Q* & *R*)’ ⊧ ‘*R* ⊃ ~*P*’

(1)	*P* ⊃ *Q*			√	Assumption
(2)	~(*Q* & *R*)			√	Assumption
(3)	~(*R* ⊃ ~*P*)			√	Negated Conclusion
(4)	*R*				3, ~⊃A
(5)	~~*P*				3, ~⊃A
(6)	*P*				5, ~~A
(7)	~*P* *	*Q*			1, ⊃A
(8)		~*Q* *	~*R* *		2, ~&A

The argument is valid because all branches close.

3.(f) ‘P’, ‘Q’, ‘$Q \supset R$’, ‘$(R \& {\sim}S) \supset {\sim}P$’ ⊧ ‘$S$’

A possible argument: “Plato writes clearly. Hume writes elegantly. If Hume writes elegantly, then so does Locke. But if this it the case and Berkeley does not write well, then Plato does not write clearly. Hence, Berkeley writes well”.

P = Plato writes well; Q = Hume writes elegantly; R = Locke writes elegantly; S = Berkeley writes well

(1)		P			Assumption
(2)		Q			Assumption
(3)		$Q \supset R$		√	Assumption
(4)		$(R \& {\sim}S) \supset {\sim}P$		√	Assumption
(5)		${\sim}S$			Negated Conclusion
(6)	~Q *		R		3, ⊃A
(7)		${\sim}(R \& {\sim}S)$	${\sim}P$ *		4, ⊃A
(8)	${\sim}R$		${\sim\sim}S$		4, ~&A
(9)	*		S *		8, ~~A

The argument is valid. Compare the tree with the truthtable: what looks more convenient? (Yes, this is a rhetorical question.)

P	Q	R	S	$((P \& Q \& (Q \supset R) \& ((R \& {\sim}S) \supset {\sim}P)) \supset S$						
T	T	T	T	T	T	T	T	F	T	**T**
T	T	T	F	T	T	T	F	T	F	**T**
T	T	F	T	T	F	F	F	F	T	**T**
T	T	F	F	T	F	F	F	F	T	**T**
T	F	T	T	F	F	T	F	F	T	**T**
T	F	T	F	F	F	T	F	T	F	**T**
T	F	F	T	F	F	T	F	F	T	**T**
T	F	F	F	F	F	T	F	F	T	**T**
F	T	T	T	F	F	T	F	F	T	**T**
F	T	T	F	F	F	T	F	T	T	**T**
F	T	F	T	F	F	F	F	F	T	**T**
F	T	F	F	F	F	F	F	F	T	**T**
F	F	T	T	F	F	T	F	F	T	**T**
F	F	T	F	F	F	T	F	T	T	**T**
F	F	F	T	F	F	T	F	F	T	**T**
F	F	F	F	F	F	T	F	F	T	**T**

3.(g) ‘$((P \supset Q) \supset (P \supset R)) \supset S$’, ‘$Q \supset R$’ ⊧ ‘$S$’

(1)	$((P \supset Q) \supset (P \supset R)) \supset S$			√	Assumption
(2)	$Q \supset R$			√	Assumption
(3)	$\sim S$				Negated Conclusion
(4)	$\sim((P \supset Q) \supset (P \supset R))$		S	√	1, ⊃A
(5)	$P \supset Q$		*	√	4, ~⊃A
(6)	$\sim(P \supset R)$			√	4, ~⊃A
(7)	P				6, ~⊃A
(8)	$\sim R$				6, ~⊃A
(9)	$\sim P$ *	Q			5, ⊃A
(10)		$\sim Q$ *	R *		2, ⊃A

All branches close: the argument is valid.

3.(h) A mother thinks, “I can get Molly to eat vegetables only if I promise to give her some chocolate later. However if I make such a promise and fail to keep it, I know she will never eat vegetables again. Obviously in order to have a healthy child I must make sure she eats vegetables. So, to have a healthy child I had better not get Molly to eat vegetables and then not give her some chocolate later.”

P = I can get Molly to eat vegetables; Q = I promise to give her some chocolate later; R = I do not keep my promise; S = I want a healthy child

‘$P \supset Q$’, ‘$(Q \,\&\, R) \supset {\sim}P$’, ‘$S \supset P$’ ⊨ ‘$S \supset (P \,\&\, {\sim}R)$’

(1)	$P \supset Q$	√	Assumption
(2)	$(Q \,\&\, R) \supset {\sim}P$	√	Assumption
(3)	$S \supset P$	√	Assumption
(4)	${\sim}(S \supset (P \,\&\, {\sim}R))$	√	Negated Conclusion
(5)	S		4, ~⊃A
(6)	${\sim}(P \,\&\, {\sim}R)$	√	4, ~⊃A
(7)	${\sim}S$ * P		3, ⊃A
(8)	${\sim}P$ * Q		1, ⊃A
(9)	${\sim}(Q \,\&\, R)$ ${\sim}P$ *		2, ⊃A
(10)	${\sim}Q$ * ${\sim}R$		9, ~&A
(11)	${\sim}P$ * ${\sim\sim}R$		6, ~&A
(12)	R *		11, ~~A

The argument is valid. (But is it a good argument?)

3.(i) “Another point to appreciate is this. The ‘incorporeal’, according to the common meaning of the word, is applied to that which can be thought of *per se*. But it is impossible to think of the incorporeal *per se* except as void. And void can neither act nor be acted upon, but merely provides bodies with motion through itself. Consequently those who say that the soul is incorporeal are talking nonsense. For if it were like that it would be unable to act and the acted upon in any way.”

Suggested Lexicon. P = The incorporeal is conceivable as such (the common view); Q = The incorporeal is conceivable as void; R = The void is causally inert; S = The soul is incorporeal (i.e. immaterial); T = The soul has causal powers (i.e. acts and is acted on)

Upshot. The argument links incorporeality with causal inertness, and uses the soul’s causal non-inerteness to argue for its non-incorporeality: the soul is a body (or a material structure of some sort).

‘$P \vee Q$’, ‘$\sim P$’, ‘$Q \supset R$’, ‘$S \supset \sim T$’, ‘T’ $\models$ ‘$\sim S$’

(1)		$P \vee Q$				√	Assumption
(2)		$\sim P$					Assumption
(3)		$Q \supset R$				√	Assumption
(4)		$S \supset \sim T$				√	Assumption
(5)		T					Assumption
(6)		S					Negated Conclusion
(7)	P *		Q				1, ∨A
(8)		$\sim Q$ *		R			3, ⊃A
(9)			$\sim S$ *		$\sim T$ *		4, ⊃A

Given these assumptions about the void and incorporeality, Epicurus’ argument is valid: all branches of the tree close.

3.(j) ‘$P \supset Q$’, ‘$Q \supset P$’, ‘$((R \supset Q) \,\&\, ({\sim}R \supset {\sim}Q))$’, ‘${\sim}R$’ ⊧ ‘${\sim}P \,\&\, {\sim}Q$’

Lexicon. P = Virtue is knowledge, Q = Virtue is teachable (virtue can be taught), R = There are teachers of virtue

(1)	$P \supset Q$	√	Prem.
(2)	$Q \supset P$	√	Prem.
(3)	$(R \supset Q) \,\&\, ({\sim}R \supset {\sim}Q)$	√	Prem.
(4)	${\sim}R$		Prem.
(5)	${\sim}({\sim}P \,\&\, {\sim}Q)$	√	Conc. (negated)
(6)	$R \supset Q$	√	3, &A
(7)	${\sim}R \supset {\sim}Q$	√	3, &A
(8)	${\sim\sim}R$ * / ${\sim}Q$		8, ⊃A
(9)	${\sim}R$ / Q *		6, ⊃A
(10)	${\sim}P$ / Q *		1, ⊃A
(11)	${\sim}Q$ / P *		2, ⊃A
(12)	${\sim\sim}P$ / ${\sim\sim}Q$		5, ~&A
(13)	P * / Q *		12, ~~A

The truth table also shows that the argument is valid: the conjunction of the premises tautologically entails the conclusion (i.e. the material implication with the conjunction of the premises as antedent and the conclusion as consequent yields a tautology).

P	Q	R	$((P \supset Q) \,\&\, (Q \supset P) \,\&\, ((R \supset Q) \,\&\, ({\sim}R \supset {\sim}Q)) \,\&\, {\sim}R) \supset ({\sim}P \,\&\, {\sim}Q)$									
T	T	T	T	T	T	T	T	T	T	F	**T**	F
T	T	F	T	T	T	T	T	F	F	F	**T**	F
T	F	T	F	F	T	F	F	F	T	F	**T**	F
T	F	F	F	F	T	F	T	F	T	F	**T**	F
F	T	T	T	F	F	F	T	F	T	F	**T**	F
F	T	F	T	F	F	F	T	F	F	F	**T**	F
F	F	T	T	T	T	F	F	F	T	F	**T**	T
F	F	F	T	T	T	T	T	T	T	T	**T**	T

Does this mean Socrates’s argument is a good one? Not necessarily. It may not be sound. But if it were *invalid*, it could not be sound at all—we would not engage it in further philosophical exploration.