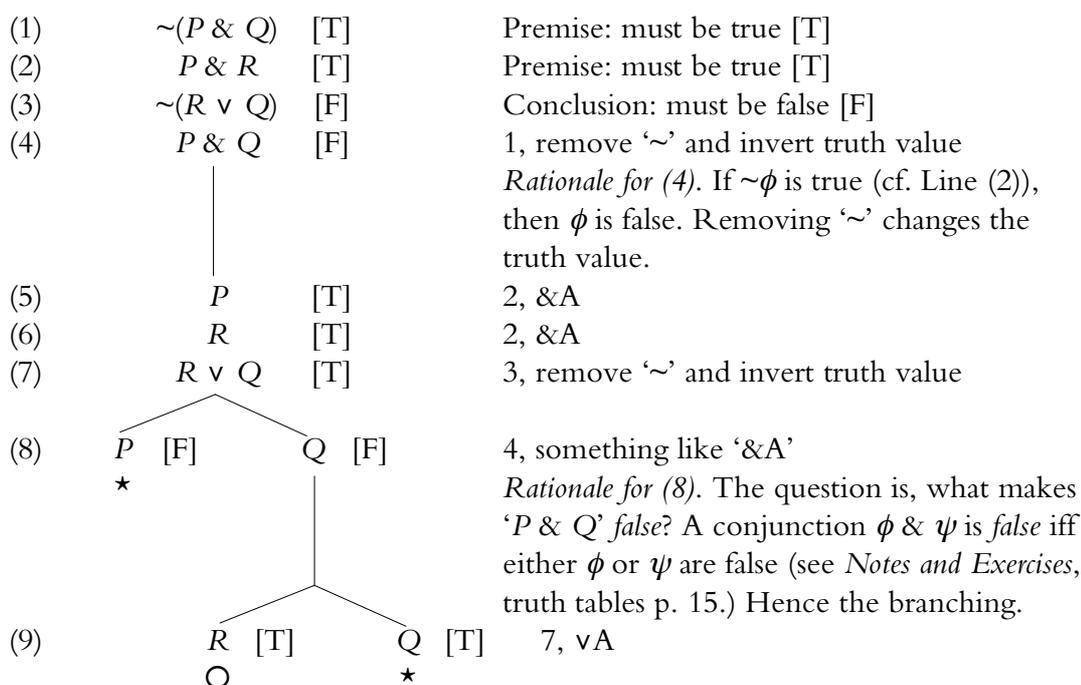


Further Remarks on PL Trees

Validity and Invalidity. An Argument A is invalid iff the set of premises Γ is true but the conclusion ϕ is false. To test validity, we check the consistency of Γ and ϕ under the assumption that the formulae in Γ are true but that ϕ is false. Hence, Γ logically entails a falsehood, so moving from the premises to the conclusion does not preserve truth. This is the guiding idea behind the tree method, which was developed in the 1950s by E. W. Beth.¹

To illustrate the basic method, here is two examples of so-called ‘signed’ trees, where each formula gets a truth value of either ‘T’ or ‘F’. This is because we assume the *falsity* of the conclusion. The tree method is thus negative or indirect: it aims to refute certain assumptions. The task is to find out whether there is a valuation under which the premises are true but the conclusion is false. If this is not possible, A is not invalid, and so valid.

*Example 1.*² ‘ $\sim(P \ \& \ Q)$ ’, ‘ $(P \ \& \ R)$ ’ \vDash ‘ $\sim(R \ \vee \ Q)$ ’

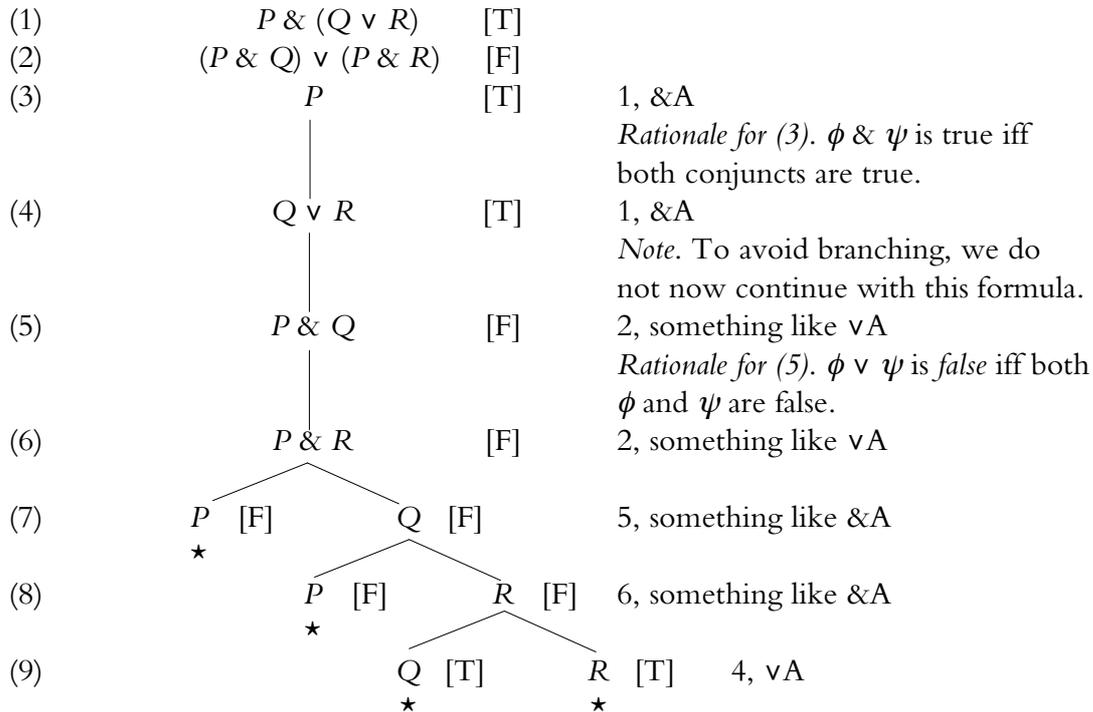


Result. There is an open branch. So there is a valuation such that the premises are true and the conclusion is false. The argument is invalid.

1 Beth, E. W. (1955). Semantic entailment and formal derivability. *Mededeelingen der Koninklijke Nederlandse Akademie van Wetenschappen, Afd. Letterkunde; nieuwe reeks*, 18, 309–42. Amsterdam: Noord-Hollandsche Uitg. Mij.

2 After Smith, P. (2003), *An Introduction to Formal Logic* (p. 150). Cambridge: Cambridge University Press.

Example 2. $(P \& (Q \vee R)) \vdash ((P \& Q) \vee (P \& R))$



Result. Every branch closes. The initial assumptions cannot be satisfied; the premise cannot be true but the conclusion false. So, the argument is valid.

However, it is simpler and more elegant to work with ‘unsigned’ trees, where *all* truth values are set to ‘true’. Given the definition of the negation, we can say: if the conclusion ϕ is false (F), then $\sim\phi$ is true (T). So, instead of assuming the falsity of the conclusion, we assume the truth of its negation. This strategy is at work in the following tree, which is Example 2 again. Is the tree different?

