Validity Revisited

Can there be valid arguments with one or more false premises? Yes.

A valid argument with false premises is possible, but it could never be sound. It may well be too that a valid argument has a false conclusion. Two examples:

(1)   All bachelors are rationalists. (F)
      All rationalists are unmarried. (F)
      Therefore, all bachelors are unmarried. (T)

(2)   All rationalists are bachelors. (F)
      All bachelors are left-handed. (F)
      Therefore, all rationalists are left-handed. (F)

Inversely, an *invalid* argument may have true premises and a true conclusion:

(3)   If Descartes is a rationalist, then so is Spinoza. (T)
      Spinoza is a rationalist. (T)
      Therefore, Descartes is a rationalist. (T)

This may seem confusing, given the typical definition of validity: if the premises of an argument A are true, then A's conclusion must be true too. This might suggest that validity is a matter of the premises' truth. But this is not the case. What is excluded is that all the premises are true while the conclusion is false. A valid argument cannot have true premises but at the same time a false conclusion. This conception is congruent with Philo's definition of a conditional: 'a sound conditional is the one which does not have a true antecedent and a false consequent'.[1]

Another Stoic definition of validity appeals more explicitly to consistency. Here is Chrysippus, another Stoic: 'a true conditional is one the contradictory of whose consequent conflicts with its antecedent'.[2] His example: 'if it is day, it is light'; hence the argument 'it is day, so it is *not* light' involves an inconsistency and is thus invalid. An argument is valid if the denial of the conclusion is incompatible with the premises. Conversely, an argument is *invalid* if the denial of the conclusion *is* compatible with the premises.[3]

A third approach to validity emphasises the 'following-from' relation, or the necessary and truth-preserving movement from A's premises to A's conclusion. This relation is 'logical consequence'. So, A is valid if and only if A's conclusion is a logical consequence of A's premises. (The symbol for logical consequence is '⊧', the

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1   ὑγιὲς δὲ συνημμένον τὸ μὴ ἀρχόμενον ἀπὸ ἀληθοῦς καὶ λῆγον ἐπὶ ψεῦδος. Long, A. A. & Sedley, D. N. (1987). *The Hellenistic Philosophers.* Cambridge: Cambridge University Press (35C). See also *Notes and Exercises*, p. 14.

2   συνημμένον οὖν ἀληθές ἐστιν οὗ τὸ ἀντικείμενον τοῦ λήγοντος μάχεται τῷ ὑγουμένῳ. (*ibid.*, 35A).

3   For a similar thought behind testing the validity of arguments, see *Notes and Exercises*, pp. 20ff.

double turnstile; see *Notes and Exercises*, §5). In a valid argument, the pattern of *inference* is such that it does not lead from truth to falsity. That is, *A* is valid if and only if the *rules* of inference are themselves valid. (This could be called a 'syntactic' approach, because it appeals to the structure of the argument.)

But these three views are merely different in their emphasis. If *A* is valid, the premises—*if* they are true at all—impose truth on the conclusion. If the premises *were* true, then the conclusion *could* not be false. Validity hence concerns the force of the 'therefore': the premises bestow their truth (if they are true) on the conclusion. (After all, *validus* means 'force' or 'strength' in Latin.) An argument is valid if there is no circumstance in which all the premises are true but not the conclusion. This is again the coherence approach: a valid argument is one in which the denial of the conclusion is inconsistent with the acceptance of the premises. The ideas of a truth-preserving inference and a 'following from' relation are in line with derivation or *deduction*: in a valid argument, the conclusion is in a sense teased out of the premises. So, if *they* are true, *it* could not be false. For in a way, the premises *contain* the conclusion, and this is also why they appear connected, and ground the support that the joint premises lend to the conclusion.

Afterthought. A special case of validity concerns tautologies and contradictions.

(4)   Spinoza proves the existence of God.
       Therefore, Plato's *Republic* is a great book or it is not.

The conclusion is a tautology, i.e. a trivially true proposition. Hence it *could* not be case that it was false, whatever the truth or falsity of the premises. This makes (4) a valid argument. But it is rather ugly, for the premise and the conclusion are unrelated, and so, there is no consequential 'force'. (The 'therefore' is vacuous.) This is the price for thinking about validity in terms of form or structure, rather than content. A similar paradox concerns the material implication (see *Notes and Exercises*, p. 14).

(5)   Locke is Oxford's finest philosopher and he is not Oxford's finest philosopher.
       Therefore, Kant never left Königsberg.

The premise is a contradiction, i.e. a trivially false proposition. Since it could not be true, there is no case in which a true premise leads to a false conclusion. So, from a contradiction, anything follows; however unrelated. But like (4), (5) is of course a rather uncompelling argument, even though it is valid. The point: a valid argument is not always a good argument.